



ТЕХНИЧЕСКИ УНИВЕРСИТЕТ – ГАБРОВО
КАТЕДРА „МАТЕМАТИКА“

СТУДЕНТСКА ОЛИМПИАДА ПО КОМПЮТЪРНА МАТЕМАТИКА

CompMath – 2011

20-22. X. 2011, Габрово

1. Calculate the value of the expression $\sqrt{\frac{x}{x+y} + \frac{y}{x-y}}$ for $x=1,1$ and $y=3,14$.

2. Calculate $\left(\frac{\sqrt{3}+5i}{4+2\sqrt{3}i}\right)^{2011}$.

3. Find the standard form of the polynomial $(x-1)^7 + (x-3)^3$.

4. Factor the polynomial $x^8 + 1$ into irreducible factors with real coefficients.

5. Find the partial fraction decomposition of the rational function

$$\frac{18x^5 + 19x^4 + 37x^3 + 166x^2 - 90x + 270}{x^6 + 2x^5 + 2x^4 + 25x^3 - 36x^2 + 78x - 72}$$

6. Solve the inequality $x+2 \geq \sqrt{x+4}$.

7. Solve in complex numbers the equation

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix} = 0.$$

8. Solve the equation $X A = B$ if $A = \begin{pmatrix} 1 & 3 & -5 \\ 8 & 6 & 9 \\ 3 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & -9 \\ 1 & 6 & 4 \\ 7 & 4 & -7 \end{pmatrix}$.

9. Solve the equation $X^2 = A$ if $A = \begin{pmatrix} 13 & 9 \\ 12 & 16 \end{pmatrix}$.

10. Solve the system

$$\begin{cases} 3x_1 - 3x_2 - x_3 - 4x_4 = 26 \\ 2x_1 + 7x_2 + 6x_3 + 15x_4 = -5 \\ 3x_1 - x_2 + 2x_3 + 6x_4 = 18 \end{cases}$$

11. Find the area of the triangle ABC if $A(45,28)$, $B(23,92)$ and $C(11,13)$.

12. Find the coordinates of the intersection point of the line $g: \frac{x-1}{1} = \frac{y+2}{6} = \frac{z-7}{8}$

and the plane $\alpha: x+2y+3z+4=0$.

13. Find the intersection points of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and the parabola $y = (x+1)^2$.

14. Find the volume of the triangle pyramid $ABCD$ with vertices $A(1,3,6)$, $B(4,7,2)$, $C(8,1,-2)$ and $D(3,2,7)$.
15. Plot the graph of the quadratic function $f(x) = ax^2 + bx + c$, given it passes through the points $A(1,2)$, $B(2,3)$ and $C(5,4)$.
16. Calculate $\lim_{n \rightarrow \infty} \frac{n^{2012} - (n - 2011)^{2012}}{2012 n^{2011}}$.
17. Calculate the third derivative $f(x) = x^{2012} \ln x$.
18. Find $f'(0)$ given $f(x) = \arctg(\sqrt{1+x} - \sqrt{1-x})$.
19. Find the values of x , such that the function $f(x) = \sqrt[3]{(x-8)^2} + \sqrt[3]{(x+1)^2}$ has a local extreme.
20. Find a function $F(x)$, which is the antiderivative of the function $f(x) = \frac{1}{x\sqrt{x^2-1}}$ in the interval $(1, \infty)$ and $F(\sqrt{2}) = 0$.
21. Calculate $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$.
22. Calculate $\int_0^1 \sqrt{x+1} \cdot \sin x^2 dx$.
23. Which are the values of $x \in (\sqrt{2}, \infty)$ such that the equality $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ holds.
24. Calculate the area of the figure, defined by the graphs of the functions $y = \sin x$ and $y = \cos 2x$ for $x \in [0, \pi]$.
25. Solve the differential equation $y' + y = x\sqrt{y}$ with initial condition $y(1) = e$.
26. Calculate the sum $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \dots + \frac{99}{100}$.
27. Calculate $\lim_{n \rightarrow \infty} \frac{\frac{1}{n+2011} + \frac{1}{n+2012} + \dots + \frac{1}{2n} - \ln 2}{\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} - \ln 2}$.
28. Solve the equation $2\arctg x + \arcsin \frac{2x}{1+x^2} = \pi$.
29. Find the real roots of the equation $x^3 - 10x^2 + 6 = 0$.
30. For which real values of the real parameter m the equation $x \ln^2 x = m$ has exactly two real roots?

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.