SECOND NATIONAL STUDENT OLYMPIAD

IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“

THE UNIVERSITY OF RUSE„ANGEL KUNCHEV“, 17-19 OCTOBER 2013

PROBLEMS FOR GROUP A

1. Calculate the value of the expression $\sqrt[3]{88,2\sqrt[4]{333-\sqrt[5]{2+4,5}}}$.

2. Find the natural numbers a and b, if their sum is 5432, and their least common multiple is 223020.

3. Factor the polynomial $f(x)=x^4-4x^3+x^2-8x-2.$ into irreducible polynomials with real coefficients.

4. Calculate z^{60}, if $z=-1+i\sqrt{3}$.

5. Find a solution of the Diophantine equation $4x+19y=5$ in the interval $(-15,5)$.

6. Find three different integers a, b and c, such that the roots of the equation $x^3+ax^2+bx+c=0$ are a, b and c.

7. The vectors $a_i=\left(\frac{1}{i},\frac{1}{i+1},\dots,\frac{1}{i+n-1}\right)$ are been constructed for $i=1,2,\dots,n$. If $a_i\cdot a_{i+1}$ is the dot product of a_i and a_{i+1}, calculate $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sum_{i=1}^{n-1}\left(a_i\cdot a_{i+1}\right)^{-1}\right]$.

8. Find the 2×2 matrices A with integer elements and a trace -1, such that $A^2+A^T=E$.

9. Solve the equation $A.X.A=B$, if $A=\begin{pmatrix}1&-2&2\\3&-4&1\\2&2&-1\end{pmatrix}$ and $B=\begin{pmatrix}-1&2&1\\1&4&-1\\2&-2&1\end{pmatrix}$.

10. Solve the system $\left|\begin{array}{l} x_1+x_2+x_3+x_4+x_5=7\\ 2x_1-x_2+3x_3-x_4-x_5=6\\ 4x_1+x_2+5x_3+x_4+x_5=20\end{array}\right.$.

11. Calculate the rank of the matrix $A=\begin{pmatrix}a&4&10&1\\3&1&1&4\\1&7&17&3\\2&2&4&1\end{pmatrix}$ depending on the values of the parameter a.

12. Let $f_1=f_2=1,\ f_{n+1}=f_n+f_{n-1}$ be Fibonacci numbers, and $Q=\begin{pmatrix}1&1\\1&0\end{pmatrix}$. Prove that $Q^{2013}=\begin{pmatrix}f_{2014}&f_{2013}\\f_{2013}&f_{2012}\end{pmatrix}$.

13. Prove that if k is a natural number, then $\operatorname{arctg}\frac{1}{2^k}+\operatorname{arctg}\frac{2^k-1}{2^k+1}=\frac{\pi}{4}$.

14. Find real numbers a, b and c such that the angles between the tangents to the graph of the function $f(x)=ax^2+bx+c$ at the points with abscises -2 and 2 with the x-axis of respectively 45° and 60°, and the greatest value of $f(x)$ over the interval $[-2,2]$ is twice as big as its smallest value over the same interval.

15. From all straight lines passing though the point $M(1,4)$, find the one that cuts from the hyperbola $xy=3$ figure with smallest area.

16. Find the length of the curve defined by the graph of the function $f(x)=\sqrt{x-x^2}+\arcsin\sqrt{x}$.

17. Find the second derivative of the function $f(x)=x^{x^x}$.

18. Find the intervals over which the function $f(x)=(x+2)e^{\frac{1}{x}}$ is continuous, find the local extremes of the function and sketch its graph over the interval $[-10,5]$.

19. Find the global extremes of the function $f(x)=\int_0^x (1-t^2)^5 dt$ over the interval $[0;2]$.

20. Calculate $\int_0^4 xf(x)\,dx,$ if the real function $y=f(x)$ satisfies the equation $y^3+3y=x.$

21. Find the local extremes of the function $f(x)=\operatorname{arctg}x-\arccos\frac{1}{\sqrt{1+x^2}}-x.$

22. Let $a<b<c<d.$ Prove that $\begin{vmatrix} \int_a^b 1dx & \int_a^b xdx & \int_a^b x^2dx \\ \int_a^c 1dx & \int_a^c xdx & \int_a^c x^2dx \\ \int_a^d 1dx & \int_a^d xdx & \int_a^d x^2dx \end{vmatrix}>0.$

23. Calculate $\int_0^{2\pi} \frac{1+\cos x}{(\sin x+\cos x+2)^3}\,dx,$ with accuracy of eight decimal points.

24. Calculate the sum $\frac{1}{2.3}-\frac{2}{3.4}+\frac{3}{4.5}-\cdots+\frac{2011}{2012.2013}$ with accuracy of six decimal points.

25. Let $S(k,n)=\sum_{i=1}^n i^k.$ Prove that $\sum_{t=0}^n \frac{S(2,3t+1)}{S(1,3t+1)}$ is always a perfect square.

26. Calculate the limit $\lim_{x\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{(n-1)\pi}{n}\right).$

27. Check if the equality $\int_0^{\pi} xf(\sin x)\,dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx,$ holds for $f(x)=\frac{x^2}{x^2+6}.$

28. Solve the differential equation $y''(x)=2y(x)+1$ with initial conditions $y(0)=1$ and $y'(0)=0.$

29. Find the smallest prime number $p>11,$ which written in decimal system contains only ones.

30. In which number system the following equality holds $14414+3403=23322$?

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.