SECOND NATIONAL STUDENT OLYMPIAD
IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“
THE UNIVERSITY OF RUSE„ANGEL KUNCHEV“, 17-19 OCTOBER 2013

PROBLEMS FOR GROUP C

1. Calculate $\sqrt{\dfrac{x+y}{x-y}}+\sqrt{\dfrac{x-y}{x+y}}$, for $x=2$ and $y=-1,5$.

2. Calculate $\sqrt{\sqrt[3]{64}\left(4+\dfrac{1}{4}\right)+1}$ and transform it into a decimal number with accuracy of ten decimal points.

3. Compare e^{π} and π^{e}.

4. Check if $7777^{2222}+2222^{7777}$ is divisible by 9.

5. Calculate $x_1^8+x_2^8-3x_1x_2$, if x_1 and x_2 are the roots of the equation $x^2-5x+11=0$.

6. Calculate $\dfrac{\left(-1+i\sqrt{3}\right)^{15}}{(1-i)^{20}}+\dfrac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$.

7. Factor the polynomial $f(x)=x^4-4x^3+x^2-8x-2.$ into irreducible multiples with real coefficients.

8. Write the polynomial $(x-3)^{12}-(x+3)^{12}$. in its standard form

9. Solve the equation $\sqrt{1+\dfrac{2}{x}}+\sqrt{\dfrac{x}{x+2}}=4.$

10. Solve the inequality $\dfrac{1}{x+1}\left(\dfrac{x-1}{x+1}+\dfrac{4x}{x^2-1}\right)\geq x.$

11. Find the number of positive roots of the equation $x^3-3\sqrt[3]{3}\,x+\sqrt{6}=0.$

12. Solve the system of equations $\begin{vmatrix} 4x & - & 3y & + & 2z & = & -4 \\ 2x & + & y & + & z & = & 3 \\ 7x & - & 2y & + & 3z & = & 0. \end{vmatrix}$

13. Calculate the determinant $\begin{vmatrix} 1+x_1 & 1 & 1 & 1 \\ 1 & 1+x_2 & 1 & 1 \\ 1 & 1 & 1+x_3 & 1 \\ 1 & 1 & 1 & 1+x_4 \end{vmatrix}$, if $x_1<x_2<x_3<x_4$ are the roots of the equation $x^4+2x^3-4x^2-2x+1=0$.

14. Given the matrix $A=\begin{pmatrix} 0 & -3 & -2 \\ 1 & -2 & 0 \\ 0 & 2 & 1 \end{pmatrix}$., calculate A^{2013}.

15. Solve the equation $A.X.A=B$, if $A=\begin{pmatrix} 1 & -2 & 2 \\ 3 & -4 & 1 \\ 2 & 2 & -1 \end{pmatrix}$ and $B=\begin{pmatrix} -1 & 2 & 1 \\ 1 & 4 & -1 \\ 2 & -2 & 1 \end{pmatrix}$.

16. Find the coordinates of the midpoint of the line obtained by crossing the circle $k: x^2+y^2=9$ with the line $l: y=2x+3$.

17. Find the area of the rectangle $ABCD$, if $A(2,1)$, $B(0,4)$ and C lies on the x-axis.

18. In a Cartesian coordinate system the plane $\alpha: x-2y+z+3=0$ and the points $A(0,-1,1)$ and $B(1,2,3)$. are given. Find the point in which the straight line AB intersects the plane α.

19. Prove that the function $f(x)=x+\sin 2x$ satisfies the equality $f''(x)+4f(x)=4x$.

20. Sketch the graph of the function $f(x)=x^4+4x^3-17118x^2+23996x+64015517$ with the local extremes of the function and its zeros.

21. Examine the function $f(x)=(x+2)e^{\frac{1}{x}}$ and sketch its graph over the interval $[-5,10]$.

22. Calculate $\int \frac{x^3}{\sqrt{\left(1+x^2\right)^3}}dx$.

23. Calculate $\int_{-1}^{1} \frac{dx}{\left(4^x+1\right)\left(x^2+4\right)}$. The result must be exact or with accuracy of at least six decimal points.

24. Check whether the equality holds $\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$, if $f(x)=\frac{x^2}{x^2+6}$.

25. Check if the equality $\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$, holds, if $f(x)=\frac{x^2}{x^2+6}$.

26. Calculate the sum $\frac{1}{1}+\frac{1}{1+2}+\cdots+\frac{1}{1+2+\cdots+n}$.

27. Calculate the limit $\lim_{x\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{(n-1)\pi}{n}\right)$.

28. Find for which values of the real parameter a the equation $\sin^4 x+\cos^4 x=a$ has a solution.

29. Prove that if k is a natural number, then $\operatorname{arctg}\frac{1}{2^k}+\operatorname{arctg}\frac{2^k-1}{2^k+1}=\frac{\pi}{4}$.

30. Consider the functions $f(x)=-x^2$ and $g(x)=a^2x^2+ax-4$, where a is a real parameter. Find the value of the parameter a, which maximizes the area of the figure constrained by the graphs of the two functions.

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.