FIRST NATIONAL STUDENT OLYMPIAD IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“

TECHNICAL UNIVERSITY - GABROVO
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Problems for group A

1. Calculate $\sqrt[3]{x+y}+\sqrt[3]{x-y}$ for $x=5{,}1$ and $y=3{,}14$.
2. Calculate $x_1^{12}+x_2^{12}$, if x_1 and x_2 are the roots of the equation $x^2-5x+11=0$.
3. Factor the polynomial $f(x)=x^4+8x^3+8x-1.$ into irreducible polynomials with real coefficients. (тук има и една точка да се махне, но не се разрешава корекцията
4. Find the least common multiple of the polynomials $f(x)$ and $g(x)$, where $f(x)=2x^4-5x^3+x^2-10x-6$ and $g(x)=x^4-7x^2-18$.
5. Find the complex roots of the equation $(x+2)^8-(x-2)^8=0$.
6. Factorize the polynomial *f(x)* into irreducible multiples over $\mathbf{Z}_3$, where $f(x)=x^9-x$.
7. Given the matrices $A=\begin{pmatrix}1&1&x\\0&1&1\\0&0&1\end{pmatrix}$ and $B=\begin{pmatrix}1&2012&x\\0&1&2012\\0&0&1\end{pmatrix}$, find the values of x for which $A^{2012}=B$?
8. Find the matrix X, if $\begin{pmatrix}1&2&3\\2&3&1\\3&1&2\end{pmatrix}X\begin{pmatrix}2&3&4\\3&4&2\\4&2&3\end{pmatrix}=\begin{pmatrix}3&4&5\\4&5&3\\5&3&4\end{pmatrix}$.
9. The sequence $\{a_n\}$ is set by the equalities $a_1=1,\ a_2=2,\ a_n=a_{n-1}-2a_{n-2}$ за for $n=3,4,\dots$. Check if the following equality holds $\begin{pmatrix}a_{102}\\a_{101}\end{pmatrix}=\begin{pmatrix}1&-2\\1&0\end{pmatrix}^{100}.\begin{pmatrix}2\\1\end{pmatrix}$. (има българско „за)
10. Examine and solve the system according to the values of the parameter λ
$$\left|\begin{array}{l}\lambda x_1+x_2+x_3+x_4=1\\ x_1+\lambda x_2+x_3+x_4=1\\ x_1+x_2+\lambda x_3+x_4=1\\ x_1+x_2+x_3+\lambda x_4=1\end{array}\right.$$
11. Find the area of ΔABC, if it's side equations are:
$$x+11y-23=0,\ 10x-y-8=0 \text{ and } 11x+10y-142=0.$$
12. Find the coordinates of the middle of the common chord of the circles
$$k_1:(x-4)^2+(y-3)^2=10 \text{ and } k_2:(x-7)^2+(y-5)^2=9.$$
13. Find the equation of the plane passing through the orthogonal projections of point $P(1,3,-2)$ onto the three coordinate planes.
14. Sketch the graph of the function $f(x)=ax^4-(a+2)x^2+3$ in $[-2,2]$, if it passes through the point $A(2,7)$.
15. Calculate $\lim\limits_{x\to\frac{\pi}{2}}(1+\operatorname{cotg}x)^{\operatorname{tg}x}$.

16. Calculate $f''\left(\frac{\pi}{6}\right)$, if $f(x)=\ln(\sin 2x+\cos 3x)$.

17. Find the greatest and smallest values of the function

$$f(x)=\begin{vmatrix} 1 & x & x & x & x \\ x & 2 & x & x & x \\ x & x & 3 & x & x \\ x & x & x & 4 & x \\ x & x & x & x & 5 \end{vmatrix} \text{ in the interval } [1;5].$$

18. Find the least positive solution of the equation $\sin x=\cos(4-x)$.

19. Find the roots of the equation $11\sin 3x=9\ln x$.

20. For which values of the parameter a the function $f(x)=\sin x+3a\sin 2x-\frac{1}{3}\sin 3x-6ax$ is increasing in the range $(-\infty;+\infty)$?

21. Find the primitive function of $f(x)=|x-1|$ in the interval $(-\infty;+\infty)$.

22. Calculate $\int_0^{\frac{\pi}{2}}\frac{dx}{1+\sin x+\cos x}$.

23. Write the Taylor series up to power ten of the function $f(t)$, which is represented as the determinant of the square matrix of order 3 $A=(a_{ij})$ with elements $a_{ij}=\int_{\frac{\pi}{4}}^{t}\frac{\cos x+\sin x}{i+j}dx$.

24. Calculate the area of the plane defined by $|x|^3+|y|^3\le 8$.

25. Find a positive integer number n, such that

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\dots+\frac{1}{n.(n+1).(n+2)}=\frac{1013545}{4054182}.$$

26. Find all natural numbers $n\in[2000;2012]$ which are solutions of the equations $\lim_{x\to\frac{\pi}{3}}(\cos x+\cos 2x+\cos 3x+\ldots+\cos nx)=\frac{1}{2}$.

27. For which real numbers a and b the equation $\int_0^{\pi}(at+bt^2)\cos mt\,dt=\frac{1}{m^2}$ holds for all positive whole numbers m?

28. Check if the function $y(x)=\frac{\cos x}{2}+\frac{e^x+e^{-x}}{4}$ is a solution of the equation $y(x)=\cos x+\int_0^x(x-t)y(t)\,dt$.

29. Find the last three digits of the decimal form of the number $\left(\ldots\left(\left(7^7\right)^7\right)\ldots\right)^7$, where there are 2012 sevens in the expression.

30. Find all four-digit numbers such that their square ends in 2016.

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.