FIRST NATIONAL STUDENT OLYMPIAD IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“

TECHNICAL UNIVERSITY – GABROVO 24-26. X. 2012

Problems for group B

1. Calculate $\sqrt[3]{x+y}+\sqrt[3]{x-y}$ for $x=5{,}1$ and $y=3{,}14$.

2. Calculate $x_1^{12}+x_2^{12}$, where x_1 and x_2 are the roots of the equation: $x^2-5x+11=0$.

3. Find the standard form of the polynomial $(x-1)(x+2)(x-3)(x+4)(x-5)(x+6)$.

4. For which values of the parameter a, zero is a double root of the polynomial

$$f(x)=x^5+(5-a)x^4-(5a+7)x^3+(7a-29)x^2+(29a+30)x-30a\,?$$

5. Solve in complex numbers the equation $x^4-5x^2+10x-6=0$.

6. Find the domain of the function $f(x)=\ln\left(e^{2x}-e^x-2\right)$.

7. Given the matrices $A=\begin{pmatrix}1&1&x\\0&1&1\\0&0&1\end{pmatrix}$ and $B=\begin{pmatrix}1&2012&x\\0&1&2012\\0&0&1\end{pmatrix}$, find values of x such that $A^{2012}=B$?

8. Find a matrix X, such that $X.\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}=\begin{pmatrix}18&24&30\\29&40&51\end{pmatrix}$.

9. The sequence $\{a_n\}$ is set by the equalities $a_1=1$, $a_2=2$, $a_n=a_{n-1}-2a_{n-2}$ за $n=3,4,\ldots$. Check if the following equality holds $\begin{pmatrix}a_{102}\\a_{101}\end{pmatrix}=\begin{pmatrix}1&-2\\1&0\end{pmatrix}^{100}.\begin{pmatrix}2\\1\end{pmatrix}$.

10. Solve the system depending on the values of the parameter α

$$\left|\begin{array}{rcrcrcl}\alpha x_1&+&x_2&+&x_3&=&1\\x_1&+&\alpha x_2&+&x_3&=&1\\x_1&+&x_2&+&\alpha x_3&=&\alpha^2\end{array}\right.$$

11. Find the area of ΔABC, if the equations of its sides are:

$$x+11y-23=0,\ 10x-y-8=0 \text{ and } 11x+10y-142=0.$$

12. Find the coordinates of the middle point of the common chord of the circumferences

$$k_1:(x-4)^2+(y-3)^2=10 \text{ and } k_2:(x-7)^2+(y-5)^2=9.$$

13. Find the equation of the plane which contains the orthogonal projections of point $P(1,3,-2)$ on the three coordinate planes.

14. The points $A(1,2,4)$, $B(5,1,3)$, $C(4,2,1)$ and $D(2,x,5)$ are the vertices of the pyramid $ABCD$. Find the values of x such that the volume of the pyramid equals 2.

15. Sketch the graph of the function $f(x)=|x-a|+|2x+b|$, if it contains the points $A(1,8)$ and $B(-3,10)$.

16. Calculate $\lim\limits_{x\to\frac{\pi}{2}}(1+\mathrm{cotg}x)^{\mathrm{tg}x}$.

17. Calculate $f''\left(\frac{\pi}{6}\right)$, if $f(x)=\ln(\sin 2x+\cos 3x)$.

18. Find the minimum and maximum of the function

$$f(x)=\begin{vmatrix} 1 & x & x & x & x \\ x & 2 & x & x & x \\ x & x & 3 & x & x \\ x & x & x & 4 & x \\ x & x & x & x & 5 \end{vmatrix} \text{ over the interval } [1;5].$$

19. Find the smallest positive solution of the equation $\sin x=\cos(4-x)$.

20. Find the roots of the equation $11\sin 3x=9\ln x$.

21. For which values of the parameter a the function $f(x)=\sin x+3a\sin 2x-\frac{1}{3}\sin 3x-6ax$ is increasing in the range $(-\infty;+\infty)$?

22. Find the antiderivative of $f(x)=|x-1|$ in the range $(-\infty;+\infty)$.

23. Calculate $\int_0^{\frac{\pi}{2}}\frac{dx}{1+\sin x+\cos x}$.

24. Calculate the area of the plane inside the curve $|x|^3+|y|^3\le 8$.

25. Find a positive integer n, such that

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\cdots+\frac{1}{n.(n+1).(n+2)}=\frac{1013545}{4054182}.$$

26. Find all natural numbers $n\in[2000;2012]$ which are solutions of the equations $\lim\limits_{x\to\frac{\pi}{3}}(\cos x+\cos 2x+\cos 3x+\ldots+\cos nx)=\frac{1}{2}$.

27. For which real numbers a and b the equation $\int_0^{\pi}(at+bt^2)\cos mt\ dt=\frac{1}{m^2}$ holds for all positive integers m?

28. Check if the function $y(x)=\frac{\cos x}{2}+\frac{e^x+e^{-x}}{4}$ is solution of the equation $y(x)=\cos x+\int_0^x(x-t)y(t)\ dt$.

29. Solve the differential equation $y''(x)-5y'(x)+6y(x)=\sin 2x+\cos x$ subject to the initial conditions $y(0)=y'(0)=0$.

30. Find all four-digit numbers such that their square ends with 2016.

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.