THIRD NATIONAL STUDENT OLYMPIAD
IN COMPUTER MATHEMATICS
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Problems for group B

1. Find a polynomial of forth degree with leading coefficient 1, which zeros are less than the zeros of $x^4+4x^3-12x^2-29x+56.$ with 3.

2. Find a polynomial of least possible degree such that for $x=1,2,3,4$ is equal respectively to $y=3,1,7,27$.

3. Find real a and $b,$ is the number $2-i$ is a root of the equation $2x^4+x^3-x^2+ax+b=0.$

4. What are the values of a for which the numbers $x,$ y and $z,$ derived from the system $\begin{vmatrix} 3x-y+z=5 \\ ax+y-z=0 \\ x+2y+az=17 \end{vmatrix}$ form an arithmetic progression in that order?

5. What are the values of the parameter a such that the system $\begin{vmatrix} ax+3y+z=0 \\ x+(a+1)y-z=0 \\ (2a-1)x+2y+4z=0 \end{vmatrix}$ has a non-zero solution?

6. Prove that if a,b,c and d are different integers, then $(a-c)^2+(b-d)^2$.is a divisor of the determinant $\begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix}.$

7. The point C lies on the line $a: x+y=2014.$ If $A(2,0)$, $B(1,4)$ and the area of the triangle ABC is $2015,$ then find the coordinates of the point $C.$

8. Find the area of the figure, defined by the curve $y=2-4x^2+4x^3-x^4,$ abscise axis and the lines $x=x_1$ and $x=x_2,$ if y has local maximum in the points x_1 and x_2.

9. Find all four-digit numbers, that are equal to the sum of the forth degrees of their digits.

10. Calculate the limit $\lim\limits_{n\to\infty}\dfrac{1}{n}\left(\dfrac{1}{2+\cos\frac{\pi}{n}}+\dfrac{1}{2+\cos\frac{2\pi}{n}}+\cdots+\dfrac{1}{2+\cos\frac{n\pi}{n}}\right).$

11. Prove the identity $\displaystyle\int_0^{\sin^2 x}\arcsin\sqrt{t}\,\mathrm{d}t+\int_0^{\cos^2 x}\arccos\sqrt{t}\,\mathrm{d}t=\frac{\pi}{4}.$

12. Compare the value of the expression $\operatorname{arctg}\frac{1}{2}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{8}$ with the number $\frac{\pi}{4}$.

13. Solve the equation $\sin x=\log_{10}x.$

14. Calculate the sum $1+\frac{x}{1!}+\frac{x^2}{2!}+\dots+\frac{x^n}{n!}+\cdots$ and compare the value of the integral $\int\limits_0^{2014} e^{-x}\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\dots+\frac{x^{2014}}{2014!}\right)dx$ with 2014.

15. Find the values of the real parameter m, for which the equation $(1-m)x^3-3mx^2-3mx+4-m=0$ has three real roots.

16. On the hyperbola $xy=-1$ the points A_n and B_n are chosen such that their abscises are respectively $\frac{n}{n+1}$ and $\frac{n+1}{n}$. Let M_n be the centre of the circle passing through the points A_n, B_n and the point $C(1,-1)$. Find the limit of the sequence of points $\{M_n\}$ for $n\to\infty$.

17. Find the intersection точка of the planes $\alpha: 7x-5y+2z-41=0$, $\beta: 4x+3y-11z+49=0$ and $\gamma: 2x+3y+4z-20=0$.

18. Find the greatest value of the function $y=2\text{tg}x-\text{tg}^2x$ in the interval $\left(0,\frac{\pi}{2}\right)$.

19. The local extreme value of the function $y=\frac{ax+b}{(x-20)(x-14)}$ is equal to 1 and is reached for $x=2014$. Find a and b and determine the kind of the extreme.

20. Solve the equation $\text{arctg}\,x-\frac{1}{2}\text{arctg}\frac{2x}{1-x^2}=\frac{\pi}{2}$.

21. Calculate the limit $\lim\limits_{n\to\infty}\left(1+\frac{1}{n^2}\right)\left(1+\frac{2}{n^2}\right)\cdots\left(1+\frac{n-1}{n^2}\right)\left(1+\frac{n}{n^2}\right)$.

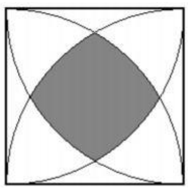
22. Find a differential equation which family of solution curves is $y=\frac{x^2-C^2}{2C}$.

23. Tangent lines to the parabola $y=-x^2+5x-6$. pass through the point $M(2,4)$. Find the area of the "curved" triangle, formed by the two tangent lines and the parabola.

24. Two circles are given. The first has a radius 3 and a centre $M(0,5)$ and the second – a radius 2 and centre $N(10,7)$. Find a point A on the first circle, a point B on the second circle and a point C on the abscise axis, such that the sum $AB+BC+AC$ is minimal. Plot the circles and the solution on one drawing.

25. Find the length of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$ and the area of the figure defined by that curve.

26. Which is the 2014-th digit of the number 2014^{2014}?



27. The vertices of a square are used as centres of four circles (but only one fourth of each one is drawn). Find the area of the shaded part of the figure.

28. A light beam starting from point to point $A(-3,0)$ is directed towards point $B(0,2)$ and is reflected by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ first at point C, and then at point D. Find the coordinates of the point D.

29. Find the volume of the body, defined by the planes $y=1$ and $z=0$, the parabolic cylinder $y=x^2$ and the paraboloid $z=x^2+y^2$.

30. Prove that the inflexion points on the graph of the function $y=\frac{x+1}{x^2+1}$ lie on a straight line.

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.