

18. Prove that if $0 < a \leq b \leq c$, then
$$\left| \begin{array}{ccc} a & b & c \\ a^3 & b^3 & c^3 \\ \frac{1}{b^3+c^3} & \frac{1}{c^3+a^3} & \frac{1}{a^3+b^3} \end{array} \right| \geq 0.$$

19. Solve the system
$$\begin{cases} 2x + (2-a)y = 6 \\ (a+1)x + y = 3 \end{cases}$$
, where a is a real parameter.

20. Find the equation of the sphere that passes through the points $A(3,1,5)$, $B(4,-8,1)$ и $C(-5,1,-3)$, if it is known that its center lies on the plane $2x + y - z - 1 = 0$.

21. For the function $f(n)$ is known that $f(1) = 2$ and $f(n) = 3f^2(n-1) - 4$ for $n > 1$. Calculate $f(1)$, $f(2)$, $f(3)$, $f(4)$ and $f(5)$.

22. Calculate the limit
$$\lim_{x \rightarrow a} \frac{x^n - a^n - n \cdot a^{n-1} \cdot (x-a)}{(x-a)^2}, n \in \mathbf{N}, a \in \mathbf{R}.$$

23. Determine the intervals of monotony and the local extrema of the function $f(x) = x^{\frac{5}{3}} - 5 \cdot x^{\frac{2}{3}}$.

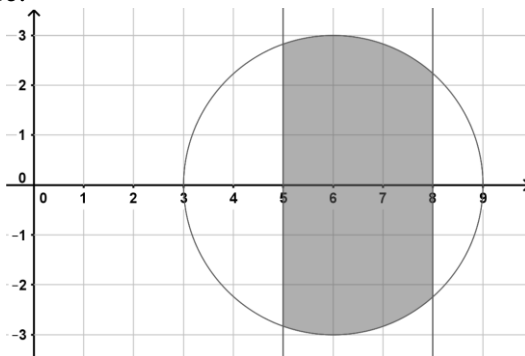
24. For which natural number n the expression $\frac{n^2}{1,001^n}$ ecomes greatest?.

25. Prove that the equality $\arctg \sqrt{\frac{1+x}{1-x}} = \frac{\pi}{4} + \frac{1}{2} \arcsin x$ holds for every $x \in (-1,1)$.

26. Calculate
$$\int_0^{2015} \frac{x}{\sin x + 2015} dx.$$

27. Calculate the length of the curve, defined by the graph of the function $y = \ln \left(\frac{e^x + 1}{e^x - 1} \right)$ in the interval $1 \leq x \leq 2$.

28. Find the area of the shaded figure.



29. Find the area of the figure, bounded by the curves кривите $y = \ln(x+2)$, $y = 2 \ln x$ and $y = 0$.

30. Let S be the area of the figure, bounded by the abscise axis and the graph of the function $f(x) = (x+2)(0,5-x)$. Find the values of the real parameter b for which the rectangular with sides $x = -2$, $y = 0$, $x = 0,5$ and $y = f(b)$ has an area S .

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.