PROBLEMS FOR GROUP A

PART 2

21. Let $S = \{1,2, \ldots ,2017\}$ be a set of integers. How many are the 2-element subsets of S, such that the product of the two integer is not divisible by 3? How many are the 3-element subsets of S, such that the product of the three integers is divisible by 4?

22. Find the smallest positive integer *n*, such that $\sum_{k=1}^{n} \frac{k}{k+1} > 2017$.

23. Find the largest integer a, such that the area of the figure bounded by the curve $y = e^{x^2}$ and the lines $x = 0, x = a, y = 0$, is smaller than 2017^{2017}.

24. Find the global extrema of the function $f(x,y) = x^2 + 4y^2$ on the closed surface determined by the inequalities $x^2 + (y+1)^2 \leq 4, y \geq -1$ и $y \leq x + 1$.

25. The function $\pi(n): \mathbb{N} \to \mathbb{N}$ gives the number of primes less than or equal to n. In 2010 Pierre Dusart proved that $\pi(n) > \frac{n}{\ln(n)-1}$ for any $x \geq N_1$ and $\pi(n) < \frac{n}{\ln(n)-1,1}$ for any $x \geq N_2$. Find the integers N_1 and N_2, if it is known that $4000 \leq N_1 \leq 6000$ and $55000 \leq N_2 \leq 65000$.

26. Evaluate $\int_1^{\infty} \frac{x}{2017^x} dx$.

27. Compute the volume of a solid figure A, such that the coordinates of its points satisfy the inequalities $x^2 + y^2 + z^2 < 1, x^2 + y^2 < z$. Is it true that the volume of A is smaller than the volume of the unit cube?

28. Solve the equation $e^{2x} + \sin(3x) = 4$.

29. Find the canonical form of the curve $c: 2x^2 + 6xy + 5y^2 + 2x - 4y + 24 = 0$ and plot its graph (using the obtained canonical equation). Compute the area of the figure bounded by the curve c.

30. Find the function $\rho = \rho(\theta)$ and plot its graph in polar coordinate system, if

$$\rho'(\theta)2^{-\theta} - \ln 2 . \cos\left(2^{\theta}\right) = 0 \text{ and } \rho\left(\log_2 \frac{\pi}{2}\right) = -0{,}7.$$