

PROBLEMS FOR GROUP A

PART 1

1. How many are the decimal digits of the integer 2017^{2017} ?
2. Calculate the expression $\sqrt{\binom{20}{17} + \binom{20}{16}}$ with 10-digits precision after the decimal point.
3. Find the rational roots of the polynomial $f(x) = 6x^4 - x^3 + 17x^2 - 3x - 3$.
4. Factor the polynomial $x^5 + 2x^4 + x^3 + 2x^2 + x - 3$ over the field \mathbb{Z}_5 .
5. For which positive integers $n < 50$ the polynomial $x^{2n} + 16x^n + 8n$ can be factorized into polynomial multipliers with integer coefficients?

6. Let A_n be the square $n \times n$ matrix $A_n = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 2 \end{pmatrix}, n \in \mathbb{N}$.

Calculate $\det A_{2017}$.

7. Solve the matrix equation $\begin{pmatrix} 3 & 5 \\ 7 & 1 \end{pmatrix} X = \begin{pmatrix} 2 & 3 & 4 & 6 \\ 10 & 15 & 20 & 30 \end{pmatrix}$.
8. Evaluate the integrals $\int \frac{x^3}{\sqrt{(1+x^2)^3}} dx$ and $\int_0^{2017} [\sqrt{x}] dx$, where $[\sqrt{x}]$ is the greatest integer that is less than or equal to $[\sqrt{x}]$.
9. Let $f(x) = \operatorname{arctg} \frac{2x}{x^2+1}$. Calculate $f''(1)$.
10. Solve the inequality $\ln \frac{1+x}{1-x} < x$.
11. Find the local extrema and plot the graph of the function $f(x) = \arcsin \frac{2x}{1+x^2}$.

12. Find the positive integer solutions of the Diophantine equation $x^3 + y^3 = (x + y)^2$.

13. The integers of the form $M_n = 2^n - 1, n \in \mathbb{N}$, are called Mersenne numbers. How many are the prime Mersenne numbers for $n \leq 2017$?

14. The sequence $\{a_n\}_{n=1}^{\infty}$ is recurrently defined by $a_1 = 2017$ and

$$a_{n+1} = \begin{cases} 3a_n + 1, & \text{if } a_n \text{ is odd;} \\ \frac{a_n}{2}, & \text{if } a_n \text{ is even.} \end{cases}$$

Find the smallest integer n , such that $a_n = 1$.

15. Find an equation of the parabola that intersects the x-axis in $x = 1$ and the y-axis in $y = -5$, and has a tangent line with equation $y = -x - 5$ in the point $x = 0$.

16. Compute the length of the arc determined by the graph of the function $y = \ln\left(\frac{e^x+1}{e^x-1}\right)$ for $1 \leq x \leq 2$.

17. Solve the system $\begin{cases} x^2 + y^2 = 1 \\ 4xy(y^2 - 1) = 0 \end{cases}$ and plot the solutions.

18. Find the coordinates of the middle of the line segment with end points that are in the intersection of the parabola $p: y = 2x^2 + 4x - 2$ and the line $l: y = x + 2$.

19. Solve the equation: $\frac{x-2017}{2018} + \frac{x-2018}{2017} = \frac{2017}{x-2018} + \frac{2018}{x-2017}$.

20. Is it true that $\frac{1}{2} + \frac{2}{3^2} + \frac{3^2}{4^3} + \dots + \frac{2016^{2015}}{2017^{2016}} > \pi$?