FIRST NATIONAL STUDENT OLYMPIAD IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“

TECHNICAL UNIVERSITY - GABROVO

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Problems for group C

1. Calculate $\sqrt[3]{\frac{x^2}{x-y}+\frac{x-y}{x+y}}$ if $x=2,3$ and $y=1,2$.

2. Calculate $x_2^5-x_1^5$, where x_1 and x_2 are the roots of the equation $x^2-2x-11=0$ and $x_1<x_2$.

3. Calculate $\frac{\left(-\sqrt{3}+i\right)^{15}}{(1-i)^{24}}$.

4. Find the standard form of the polynomial $(x-2)^9-(x+5)^5$.

5. Factor the polynomial x^8-16. to factors with real coefficients.

6. Solve the equation $32x^4-128x^3+114x^2+63x-81=0$.

7. Solve the inequality $\sqrt{5-2x}<6x-1$.

8. Find the domain of the function $f(x)=\sqrt{e^{2x}-e^x-6}$.

9. Solve the system $\begin{vmatrix} x & + & 2y & - & z & = & -3 \\ 2x & + & 3y & + & z & = & -1 \\ x & - & y & - & z & = & 3 \end{vmatrix}$

10. Calculate the determinant $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$.

11. Solve the equation $\begin{vmatrix} x & 1 & 2 \\ 3 & x & 4 \\ 5 & 6 & x \end{vmatrix}=20$.

12. For which values of a and c the following equality holds: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.\begin{pmatrix} a & 6 \\ c & 7 \end{pmatrix}=\begin{pmatrix} a & 6 \\ c & 7 \end{pmatrix}.\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$?

13. Sketch the graph of the function $f(x)=x^2+bx+c$, if it contains the points $A(2,4)$ and $B(5,1)$.

14. Calculate $\lim\limits_{x\to\infty}\arcsin\frac{1-x^2}{1+x^2}$.

15. Find the derivative of the function $f(x)=e^{\sqrt{x}}\sin^2 x$.

16. Find $f''\left(\frac{1}{4}\right)$, for $f(x)=\arccos\sqrt{x}$.

17. Define the intervals over which the function $f(x)=x-\sqrt[3]{x^2}$. is increasing and decreasing.

18. Find the local extremes of the function $f(x) = -3x^4 + 20x^3 - 35x^2 + 24.$

19. Find the maximum of the function $f(x) = \dfrac{\ln x - 1}{x}$ over the interval $(0; +\infty)$.

20. Determine the number of the positive roots of the equation $x^3 - 14x + 11 = 0.$

21. Find the roots of the equation $\sin 2x = x.$

22. For which values of the real parameter a the equation $\dfrac{x^2 - x + 1}{x^2 + x + 1} = a$ has a real solution?

23. Calculate $\displaystyle\int \frac{dx}{(x+1)\sqrt{1-x^2}}.$

24. Calculate $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx.$

25. Calculate the area bounded by the graphs of functions $f(x) = 3x - 4x^3$ and $g(x) = -x^3$ for $x \geq 0.$

26. Calculate the sum $\dfrac{1^2}{2^3} + \dfrac{2^2}{3^3} + \cdots + \dfrac{19^2}{20^3}.$

27. Find a positive integer $n,$ such that

$$\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \cdots + \frac{1}{n.(n+2)} = \frac{3038623}{4054182}.$$

28. Calculate the sum $\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \cdots + \dfrac{1}{n^4} + \cdots$

29. Check if the function $y(x) = \mathrm{e}^x - 1$ is a solution to the equation $y''(x) + \displaystyle\int_0^x \mathrm{e}^{2(x-t)} y'(t)\,dt = \mathrm{e}^{2x}.$

30. Solve the differential equation $y''(x) - 3y'(x) + 2y(x) = 0$ subject to the initial conditions $y(0) = 3,$ $y'(0) = 2.$

Each problem is worth 2 points.
All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.