SECOND NATIONAL STUDENT OLYMPIAD

IN COMPUTER MATHEMATICS „ACADEMICIAN STEFAN DODUNEKOV“

THE UNIVERSITY OF RUSE„ANGEL KUNCHEV“, 17-19 OCTOBER 2013

PROBLEMS FOR GROUP B

1. Calculate $\sqrt{\frac{x+y}{x-y}}+\sqrt{\frac{x-y}{x+y}}$, for $x=2$ and $y=-1,5$.

2. Find the value of $x_1^8+x_2^8-3x_1x_2$, if x_1 and x_2 are the roots of the equation $x^2-5x+11=0$.

3. Simplify the expression $\frac{2-x}{x+1}\cdot\frac{3x^4-24x^3-3x^2+204x-252}{220x-70x^2-168-15x^3+10x^4-x^5}$.

4. Factor the polynomial $f(x)=x^4-4x^3+x^2-8x-2.$ into irreducible multiples with real coefficients.

5. Find for which integers n the value of the expression $\frac{n^4+2n^3-3n^2+n-94}{n-3}$ is an integer.

6. Given the matrix $A=\begin{pmatrix}0 & -3 & -2\\ 1 & -2 & 0\\ 0 & 2 & 1\end{pmatrix}$., calculate A^{2013}.

7. Solve the equation $\begin{pmatrix}1 & -2 & 2\\ 3 & -4 & 1\\ 2 & 2 & -1\end{pmatrix}.X.\begin{pmatrix}1 & -2 & 2\\ 3 & -4 & 1\\ 2 & 2 & -1\end{pmatrix}=\begin{pmatrix}-1 & 2 & 1\\ 1 & 4 & -1\\ 2 & -2 & 1\end{pmatrix}$.

8. Find the rank of the matrix $A=\begin{pmatrix}a & 4 & 10 & 1\\ 3 & 1 & 1 & 4\\ 1 & 7 & 17 & 3\\ 2 & 2 & 4 & 1\end{pmatrix}$ according to the values of the parameter a.

9. Given the points $A(2,-1)$ and $B(1,0)$., find the geometric set of points in the plane, that could be point C of a triangle ABC so that its area equals 10.

10. In a Cartesian coordinate system the plane $\alpha: x-2y+z+3=0$ and the points $A(0,-1,1)$ and $B(1,2,3)$. are given. Find the point in which the straight line AB intersects the plane α.

11. The sequence $\{a_n\}$ is set by the equalities $a_1=1,\ a_2=0,\ 2a_{n+1}=2a_n+a_{n+2}$ for $n\geq 1$. Calculate a_{2013}.

12. Find three different integers a, b and c, such that the roots of the equation $x^3+ax^2+bx+c=0$ are a, b and c.

13. Let $f(x)=x^{x^x}$, find $f'(2)$ with accuracy of four decimal points.

14. Prove that the equation $\frac{2x}{1+x^4}+\cos x=\frac{\pi}{4}+\sin 1$ has a solution in the interval $(0,1)$.

15. Prove that if k is a natural number, then $\operatorname{arctg}\frac{1}{2^k}+\operatorname{arctg}\frac{2^k-1}{2^k+1}=\frac{\pi}{4}$.

16. Sketch the graph of the function $f(x)=x^4+4x^3-17118x^2+23996x+64015517$ with the local extremes of the function and its zeros.

17. Find the local extremes of the function $f(x)=\operatorname{arctg}x-\arccos\frac{1}{\sqrt{1+x^2}}-x$.

18. Find real numbers a, b and c such that the angles between the tangents to the graph of the function $f(x)=ax^2+bx+c$ at the points with abscises -2 and 2 with the x-axis of respectively 45° and 60°, and the greatest value of $f(x)$ over the interval $[-2,2]$ is twice as big as its smallest value over the same interval.

19. Find the length of the curve defined by the graph of the function $f(x)=\sqrt{x-x^2}+\arcsin\sqrt{x}.$

20. Calculate the area of the tetragon, determined by the curves of the ellipses $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $\frac{x^2}{4}+\frac{y^2}{9}=1.$

21. Calculate $\int_{-1}^{1}\frac{dx}{(4^x+1)(x^2+4)}$. The result must be exact or with accuracy of at least six decimal points.

22. Check if the equality $\int_0^{\pi} xf(\sin x)\,dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx,$ holds, if $f(x)=\frac{x^2}{x^2+6}.$

23. Let $a<b<c<d.$ Prove that the inequality holds $\begin{vmatrix}\int_a^b 1dx & \int_a^b xdx & \int_a^b x^2dx\\ \int_a^c 1dx & \int_a^c xdx & \int_a^c x^2dx\\ \int_a^d 1dx & \int_a^d xdx & \int_a^d x^2dx\end{vmatrix}>0.$

24. Calculate $\int_0^4 xf(x)\,dx,$ if the real function $y=f(x)$ satisfies the equation $y^3+3y=x.$

25. Find the global extremes of the function $f(x)=\int_0^x (1-t^2)^5 dt$ over the interval $[0;2].$

26. From all straight lines passing though the point $M(1,4),$ find the one that cuts the figure with the smallest area from the hyperbola $xy=3$.

27. Calculate the sum $\frac{1}{1}+\frac{1}{1+2}+\cdots+\frac{1}{1+2+\cdots+n}.$

28. Calculate the limit $\lim_{x\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{(n-1)\pi}{n}\right).$

29. The vectors $a_i=\left(\frac{1}{i},\frac{1}{i+1},\ldots,\frac{1}{i+n-1}\right)$ are been constructed for $i=1,2,\ldots,n$. If $a_i\cdot a_{i+1}$ is the dot product of a_i and a_{i+1}, calculate $\lim_{n\to\infty}\left[\frac{1}{n^2}\sum_{i=1}^{n-1}(a_i\cdot a_{i+1})^{-1}\right].$

30. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sum_{i=1}^{n-1}(a_i\cdot a_{i+1})^{-1}\right].$

31. Sketch the graph of the function $y(x),$ which is the solution of the differential equation $y''(x)=2y(x)+1$ with initial conditions $y(0)=1$ and $y'(0)=0.$

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.