1. Calculate $\sqrt{\frac{x+y}{x-y}} + \sqrt{\frac{x-y}{x+y}}$, for $x=2$ and $y=-1.5$.

2. Find the value of $x_1^2+x_2^2-3x_1x_2$, if $x_1$ and $x_2$ are the roots of the equation $x^2-5x+11=0$.

3. Simplify the expression $\frac{2-x}{x+1} \cdot \frac{3x^4-24x^3-3x^2+204x-252}{220x-70x^2-168-15x^3+10x^4-x^5}$.

4. Factor the polynomial $f(x) = x^4-4x^3+x^2-8x-2$. into irreducible multiples with real coefficients.

5. Find for which integers $n$ the value of the expression $\frac{n^4+2n^3-3n^2+n-94}{n-3}$ is an integer.

6. Given the matrix $A = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -2 & 0 \\ 2 & 1 & 0 \end{pmatrix}$, calculate $A^{2013}$.

7. Solve the equation $\begin{pmatrix} 1 & -2 & 2 \\ 3 & -4 & 1 \\ 2 & 2 & -1 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 1 & -2 & 2 \\ 3 & -4 & 1 \\ 2 & 2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 4 & -1 \\ 2 & -2 & 1 \end{pmatrix}$.

8. Find the rank of the matrix $A = \begin{pmatrix} a & 4 & 10 & 1 \\ 3 & 1 & 1 & 4 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 1 \end{pmatrix}$ according to the values of the parameter $a$.

9. Given the points $A(2,-1)$ and $B(1,0)$, find the geometric set of points in the plane, that could be point $C$ of a triangle $ABC$ so that its area equals 10.

10. In a Cartesian coordinate system the plane $\alpha: x-2y+z+3=0$ and the points $A(0,-1,1)$ and $B(1,2,3)$ are given. Find the point in which the straight line $AB$ intersects the plane $\alpha$.

11. The sequence $\{a_n\}$ is set by the equalities $a_1 = 1$, $a_2 = 0$, $2a_{n+1} = 2a_n + a_{n+2}$ for $n \geq 1$. Calculate $a_{2013}$.

12. Find three different integers $a$, $b$ and $c$, such that the roots of the equation $x^3+ax^2+bx+c=0$ are $a$, $b$ and $c$.

13. Let $f(x) = x^x$, find $f''(2)$ with accuracy of four decimal points.

14. Prove that the equation $\frac{2x}{1+x^3} + \cos x = \frac{\pi}{4} + \sin 1$ has a solution in the interval $(0,1)$.

15. Prove that if $k$ is a natural number, then $\arctg \frac{1}{2^k} + \arctg \frac{2^k-1}{2^k+1} = \frac{\pi}{4}$.

16. Sketch the graph of the function $f(x) = x^4 + 4x^3 - 17118x^2 + 23996x + 64015517$ with the local extremes of the function and its zeros.

17. Find the local extremes of the function $f(x) = \arctg x - \arccos \frac{1}{\sqrt{1+x^2}} - x$.

18. Find real numbers $a$, $b$ and $c$ such that the angles between the tangents to the graph of the function $f(x) = ax^2 + bx + c$ at the points with abscises $-2$ and $2$ with the $x$-axis of respectively $45^\circ$ and $60^\circ$, and...
the greatest value of $f(x)$ over the interval $[-2, 2]$ is twice as big as its smallest value over the same interval.

19. Find the length of the curve defined by the graph of the function $f(x) = \sqrt{x - x^2} + \arcsin \sqrt{x}$.

20. Calculate the area of the tetragon, determined by the curves of the ellipses $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

21. Calculate $\int_{-1}^{1} \frac{dx}{(4^x + 1)(x^2 + 4)}$. The result must be exact or with accuracy of at least six decimal points.

22. Check if the equality $\int_{0}^{\pi} \int_{0}^{\pi} f(x) \sin(x) \, dx = \frac{\pi}{2} \int_{0}^{\pi} f(\sin x) \, dx$, holds, if $f(x) = \frac{x^2}{x^2 + 6}$.

23. Let $a < b < c < d$. Prove that the inequality holds $\int_{a}^{d} (b - a) dx > 0$.

24. Calculate $\int_{0}^{\pi} xf(x) \, dx$, if the real function $y = f(x)$ satisfies the equation $y^3 + 3y = x$.

25. Find the global extremes of the function $f(x) = \int_{0}^{x} (1 - t^2)^5 \, dt$ over the interval $[0; 2]$.

26. From all straight lines passing through the point $M(1, 4)$, find the one that cuts the figure with the smallest area from the hyperbola $xy = 3$.

27. Calculate the sum $\frac{1}{1} + \frac{1}{1+2} + \cdots + \frac{1}{1+2+\cdots+n}$.

28. Calculate the limit $\lim_{x \to \infty} \frac{1}{n} \left( \sin \frac{n \pi}{n} + \sin \frac{2n \pi}{n} + \cdots + \sin \left( \frac{(n-1)\pi}{n} \right) \right)$.

29. The vectors $a_i = \left( \frac{1}{i}, \frac{1}{i+1}, \ldots, \frac{1}{i+n-1} \right)$ are constructed for $i = 1, 2, \ldots, n$. If $a_i \cdot a_{i+1}$ is the dot product of $a_i$ and $a_{i+1}$, calculate $\lim_{n \to \infty} \frac{1}{n^2} \sum_{i=1}^{n-1} (a_i \cdot a_{i+1})^{-1}$.

30. $\lim_{n \to \infty} \frac{1}{n^2} \sum_{i=1}^{n-1} (a_i \cdot a_{i+1})^{-1}$.

31. Sketch the graph of the function $y(x)$, which is the solution of the differential equation $y''(x) = 2y(x) + 1$ with initial conditions $y(0) = 1$ and $y'(0) = 0$.

Each problem is worth 2 points.

All numerical calculations must be performed with the expected computing mathematical accuracy for the corresponding computer algebra system.