PROBLEMS FOR GROUP A

PART 2

21. Let \( S = \{1, 2, \ldots, 2017\} \) be a set of integers. How many are the 2-element subsets of \( S \), such that the product of the two integer is not divisible by 3? How many are the 3-element subsets of \( S \), such that the product of the three integers is divisible by 4?

22. Find the smallest positive integer \( n \), such that \( \sum_{k=1}^{n} \frac{k}{k+1} > 2017 \).

23. Find the largest integer \( a \), such that the area of the figure bounded by the curve \( y = e^{x^2} \) and the lines \( x = 0 \), \( x = a \), \( y = 0 \), is smaller than 2017.

24. Find the global extrema of the function \( f(x, y) = x^2 + 4y^2 \) on the closed surface determined by the inequalities \( x^2 + (y + 1)^2 \leq 4 \), \( y \geq -1 \) and \( y \leq x + 1 \).

25. The function \( \pi(n): \mathbb{N} \to \mathbb{N} \) gives the number of primes less than or equal to \( n \). In 2010 Pierre Dusart proved that \( \pi(n) > \frac{n}{\ln(n) - 1} \) for any \( x \geq N_1 \) and \( \pi(n) < \frac{n}{\ln(n) - 1} \) for any \( x \geq N_2 \). Find the integers \( N_1 \) and \( N_2 \), if it is known that \( 4000 \leq N_1 \leq 6000 \) and \( 55000 \leq N_2 \leq 65000 \).

26. Evaluate \( \int_1^{\infty} \frac{x}{2017^x} \, dx \).

27. Compute the volume of a solid figure \( A \), such that the coordinates of its points satisfy the inequalities \( x^2 + y^2 + z^2 < 1 \), \( x^2 + y^2 < z \). Is it true that the volume of \( A \) is smaller than the volume of the unit cube?

28. Solve the equation \( e^{2x} + \sin(3x) = 4 \).

29. Find the canonical form of the curve \( c: 2x^2 + 6xy + 5y^2 + 2x - 4y + 24 = 0 \) and plot its graph (using the obtained canonical equation). Compute the area of the figure bounded by the curve \( c \).

30. Find the function \( \rho = \rho(\theta) \) and plot its graph in polar coordinate system, if

\[
\rho'(\theta)2^{-\theta} - \ln2 \cdot \cos(2^\theta) = 0 \quad \text{and} \quad \rho\left(\log_2 \frac{\pi}{2}\right) = -0.7.
\]