EIGHTH NATIONAL STUDENT OLYMPIAD

IN COMPUTER MATHEMATICS „PROFESSOR STEFAN DODUNEKOV"

TECHNICAL UNIVERSITY OF SOFIA

8-10 NOVEMBER 2019

==

PROBLEMS FOR GROUP A

1. Find all primes in the sequence $\{20^n+19^n\}$, n=1, 2, 3,…, 2019.

2. Find the solutions ($0<x,y,z<5$) of the equation $x^{19}+y^{19} = z^{19}$ (mod 5).

3. Find the maximal value of the function $f(x)=\begin{vmatrix} 2 & x & x^2 & x^3 \\ x & 0 & x & x^2 \\ x^2 & x & 1 & x \\ x^3 & x^2 & x & 9 \end{vmatrix}$ in the interval [0, 2019].

4. From the first 2019 Fibonacci numbers, find the number of those containing the sequence '2019' in their decimal form.

5. Solve the equation $e^{20x}+19\sin(x)=2019$.

6. Find the integers $n<51$ such that the polynomial $x^n + 2048$ can be decomposed into irreducible factors with integer coefficients (it is not irreducible over the integers).

7. What is the smallest integer k such that 2020_k (base k positional numeral system) does not have different digits (has the form xx..x) as a decimal?

8. Calculate the area of the region bounded by the curve $x^{20}+y^{20}=1$. Is it true that the figure covers over 99% of the square with side length 2 that contains it?

9. Solve the system of the equations $x^y = 2018, y^x = 2019$.

10. Calculate $\sqrt{2019+\sqrt[3]{2019}}+\sqrt[3]{2019+\sqrt{2019}}$ with 20 digits precision.

11. Find all 5-digit integers $N = \overline{abcde}$, whose digits satisfy the equation $a^4 + b^4 + c^4 + d^4 + e^4 = 2019$ and $a < b < c < d < e$.

12. Find the real numbers a,b,c,d, such that

$$2x^5 - 2\sqrt{5}x - 2x + \sqrt{5} - 5 = 2(x^2 - x + a)(x^3 + bx^2 + cx + d)$$

is true for any real x.

13. Calculate the integral $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-x^2-y^2}\,dxdy$.

14. Calculate $\lim_{n\to\infty}\int_0^1 x\sqrt{x\sqrt[3]{x\sqrt[4]{x\sqrt[5]{x \ldots x\sqrt[n]{x}}}}}\,dx$.

15. Find the real numbers a, b and c such that the equality $\int_0^{\pi}(ax + bx^2 + cx^3)\sin(mx)dx = \frac{1}{m^3}$ is true for any positive integer m .

16. Find the number of all 3-element subsets of S = {1, 2, 3, ..., 219} such that the sum of the three integers is a prime. Which of those primes has a maximal sum of its digits?

17. Calculate the length of the curve $y = 1 - \ln(\cos x)$ for $x \in [0, \frac{\pi}{6}]$.

18. Calculate $\lim_{x \to 2} \left(\frac{x}{2}\right)^{\frac{1}{x-2}}$.

19. Find the minimum distance from a point on the sphere with equation $(x-12)^2 + (y-11)^2 + (z-10)^2 = 9$ to the plane passing through the point $A(1,2,0)$ and perpendicular to the vector $\vec{v}(1,1,1)$.

20. Plot the curves given by the equations $x^4 - x^2 + \mathrm{y}^2 = 0$ and $y^4 - y^2 + \mathrm{x}^2 = 0$, and calculate the area of the region bounded by these curves.

21. How many are the 3-digit primes formed by three consecutive digits in the first 2019 digits after the decimal point of the constant e?

22. Find the number of the intersection points of the circle $c: x^2 + y^2 = 36$ and the ellipse $e: \frac{(x+1)^2}{a^2} + \frac{y^2}{36} = 1$ in dependence on the values of the parameter $a, a \neq 0$.

23. Given the matrices $\mathrm{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 0 & 8 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & -4 & -3 \\ -2 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, find a matrix C such that $A.B = A^{-1}.C^{-1}.B^{-1}$.

24. Calculate $\sqrt{1 + \sqrt[3]{2 + \sqrt[4]{3 + \sqrt[5]{4 + \cdots \sqrt[2020]{2019}}}}}$.

25. Find the volume of the solid of revolution obtained by rotating of the graph of the function $y = f(x) = \ln(x^2)$, $x \in (0,1)$ about the y-axis. Plot the obtained solid of revolution.

26. How many positive numbers are there among the first 1000 members of the sequence $\{\cos(5^k), k \in \mathbb{N}\}$?

27. Find the smallest 5-digit prime *p* such that *p+2* is also prime.

28. Calculate the sum $\sum_{\kappa=-\infty}^{+\infty} \frac{\kappa}{2^{|k|}} \cdot \left(\frac{5}{3}\right)^k$.

29. The tetrahedron ABCD has volume V= 5m^3, and three of its vertices are A(2,1,-1), B(3,0,1) and C(2,-1,3). The fourth vertex D belongs to the y-axes. Find the coordinates of D and the height from the vertex D.

30. Generate a list consisting of all 3-digit positive integers divisible by 51, written in octal number system.