PROBLEMS FOR GROUP A

1. Find all primes in the sequence \( \{20^n+19^n\} \), \( n=1, 2, 3,..., 2019 \).

2. Find the solutions \((0<x,y,z<5)\) of the equation \( x^{19}+y^{19}=z^{19} \pmod{5} \).

3. Find the maximal value of the function \( f(x) = \begin{vmatrix} 2 & x & x^2 & x^3 \\ x & 0 & x & x^2 \\ x^2 & x & 1 & x \\ x^3 & x^2 & x & 9 \end{vmatrix} \) in the interval \([0, 2019]\).

4. From the first 2019 Fibonacci numbers, find the number of those containing the sequence ‘2019’ in their decimal form.

5. Solve the equation \( e^{20x}+19\sin(x)=2019 \).

6. Find the integers \( n<51 \) such that the polynomial \( x^n + 2048 \) can be decomposed into irreducible factors with integer coefficients (it is not irreducible over the integers).

7. What is the smallest integer \( k \) such that \( 2020^k \) (base \( k \) positional numeral system) does not have different digits (has the form \( xx...x \)) as a decimal?

8. Calculate the area of the region bounded by the curve \( x^{20}+y^{20}=1 \). Is it true that the figure covers over 99% of the square with side length 2 that contains it?

9. Solve the system of the equations \( x^y = 2018, y^x = 2019 \).

10. Calculate \( \sqrt{2019} + \sqrt[3]{2019} + \sqrt[3]{2019} + \sqrt[3]{2019} \) with 20 digits precision.

11. Find all 5-digit integers \( N = abcde \), whose digits satisfy the equation \( a^4 + b^4 + c^4 + d^4 + e^4 = 2019 \) and \( a < b < c < d < e \).

12. Find the real numbers \( a,b,c,d \), such that

\[
2x^5 - 2\sqrt{5}x - 2x + \sqrt{5} - 5 = 2(x^2 - x + a)(x^3 + bx^2 + cx + d)
\]

is true for any real \( x \).

13. Calculate the integral \( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} \, dx \, dy \).

14. Calculate \( \lim_{n \to \infty} \int_{0}^{1} x \sqrt[2^n]{x} \sqrt[2^n]{x} \sqrt[2^n]{x} \ldots \sqrt[2^n]{x} \, dx \).
15. Find the real numbers $a$, $b$, and $c$ such that the equality \( \int_0^\pi (ax + bx^2 + cx^3) \sin(mx) \, dx = \frac{1}{m^2} \) is true for any positive integer $m$.

16. Find the number of all 3-element subsets of $S = \{1, 2, 3, \ldots, 219\}$ such that the sum of the three integers is a prime. Which of those primes has a maximal sum of its digits?

17. Calculate the length of the curve $y = 1 - \ln(\cos x)$ for $x \in [0, \frac{\pi}{6}]$.

18. Calculate $\lim_{x \to 2} \left( \frac{x}{2} \right)^{\frac{1}{x^2}}$.

19. Find the minimum distance from a point on the sphere with equation $(x - 12)^2 + (y - 11)^2 + (z - 10)^2 = 9$ to the plane passing through the point $A(1,2,0)$ and perpendicular to the vector $\vec{v}(1,1,1)$.

20. Plot the curves given by the equations $x^4 - x^2 + y^2 = 0$ and $y^4 - y^2 + x^2 = 0$, and calculate the area of the region bounded by these curves.

21. How many are the 3-digit primes formed by three consecutive digits in the first 2019 digits after the decimal point of the constant $e$?

22. Find the number of the intersection points of the circle $c: x^2 + y^2 = 36$ and the ellipse $e: \frac{(x+1)^2}{a^2} + \frac{y^2}{36} = 1$ in dependence on the values of the parameter $a, a \neq 0$.

23. Given the matrices $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & -4 & -3 \\ -2 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, find a matrix $C$ such that $A.B = A^{-1}.C^{-1}.B^{-1}$.

24. Calculate $\sqrt{1 + \sqrt{\frac{3}{2 + \sqrt{\frac{5}{3 + \sqrt{\cdots + \frac{2928}{\sqrt{2019}}}}}}}}$.

25. Find the volume of the solid of revolution obtained by rotating of the graph of the function $y = f(x) = \ln(x^2)$, $x \in (0,1)$ about the $y$-axis. Plot the obtained solid of revolution.

26. How many positive numbers are there among the first 1000 members of the sequence $\{\cos(5^k), k \in \mathbb{N}\}$?

27. Find the smallest 5-digit prime $p$ such that $p+2$ is also prime.

28. Calculate the sum $\sum_{k=-\infty}^{+\infty} \frac{k}{2|k|} \left( \frac{5}{3} \right)^k$.

29. The tetrahedron $ABCD$ has volume $V = 5m^3$, and three of its vertices are $A(2,1,-1)$, $B(3,0,1)$ and $C(2,-1,3)$. The fourth vertex $D$ belongs to the $y$-axes. Find the coordinates of $D$ and the height from the vertex $D$.

30. Generate a list consisting of all 3-digit positive integers divisible by 51, written in octal number system.